Kink-induced symmetry breaking patterns in brane-world $SU(3)^3$ trinification models.

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The trinification grand unified theory (GUT) has gauge group $SU(3)^3$ and a discrete symmetry permuting the $SU(3)$ factors. In common with other GUTs, the attractive nature of the fermionic multiplet assignments is obviated by the complicated multi-parameter Higgs potential apparently needed for phenomenological reasons, and also by vacuum expectation value (VEV) hierarchies *within* a given multiplet. This motivates the rigorous consideration of Higgs potentials, symmetry breaking patterns and alternative symmetry breaking mechanisms in models with this gauge group. Specifically, we study the recently proposed "clash of symmetries" brane-world mechanism to see if it can help with the symmetry breaking conundrum. This requires a detailed analysis of Higgs potential global minima and kink or domain wall solutions interpolating between the disconnected global minima created through spontaneous discrete symmetry breaking. Sufficiently long-lived metastable kinks can also be considered. We develop what we think is an interesting, albeit speculative, brane-world scheme whereby the hierarchical symmetry breaking cascade, trinification to left-right symmetry to the standard model to colour cross electromagnetism, may be induced without an initial hierarchy in vacuum expectation values. Another motivation for this paper is simply to continue the exploration of the rich class of kinks arising in models that are invariant under both discrete and continuous symmetries.

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I. INTRODUCTION

Symmetry principles are basic to quantum field theoretic models of particle physics and to general relativity. In particle physics, the fundamental fields are placed into representations of a gauge group and then an invariant Lagrangian is constructed from the fields and their 4-gradients. In some models, global symmetries are also imposed to either forbid unwanted terms or to relate otherwise independent parameters. The standard model (SM) Lagrangian is defined as the most general linear combination of all $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge-invariant renormalisable terms constructed from the familiar quark, lepton, gauge and Higgs boson fields. The Higgs doublet self-interactions cause the ground or vacuum state of the SM to respect the smaller gauge group $SU(3)_c \otimes U(1)_Q$, where Q is electric charge. This electroweak spontaneous symmetry breaking (SSB) gives masses to the W and Z bosons and the quarks and leptons. One of the theoretically unsatisfactory features of the SM arises from the Higgs sector: the proliferation of Yukawa coupling constants and hence the *a priori* arbitrary nature of the fermion masses. The SM Higgs potential, on the other hand, is rather appealing: it is very simple, having only two free parameters, and it is impossible to spontaneously break electromagnetic and colour gauge invariance.

Much effort has been expended on SM extensions where the new gauge groups are larger than $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, the goal being to construct more predictive models while retaining the phenomenological successes of the SM. Unfortunately, no one has yet constructed a phenomenologically successful extension that has fewer parameters than the SM itself![1] The basic problem is that the larger the symmetry of the Lagrangian, the more SSB is required. The

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[1] Unless one starts with the default extension to the SM featuring nonzero neutrino masses, namely the see-saw model with three right-handed neutrinos. There are successful models employing leptonic family symmetries that have fewer parameters [1].

resulting large Higgs sectors of almost all extended models are necessarily full of arbitrary parameters, with the Higgs potentials often too complicated to properly analyse. This motivates the search for alternative SSB mechanisms. The purpose of this paper is to study spontaneous symmetry breaking in trinification-style grand unified theories (GUTs) [2, 3, 4, 5].

A new mechanism called the "clash of symmetries" [6, 7] was recently proposed within the brane-world context [8] (for a general review of brane-world models, see [9, 10] and references therein). It still uses elementary scalar fields, but relaxes the requirement that the background Higgs fields correspond to the spatially homogeneous configurations given by a single global minimum of the Higgs potential. Instead, the stable configurations corresponding to topological kinks or domain walls are used, where the 3-brane constituting our universe is located at the coincident centres of the walls. (In fact, it is well-known that branes can be dynamically generated through the interactions between scalar fields and higher-dimensional gravity endowed with a cosmological term [11].) The spontaneous breakdown pattern now becomes a function of the extra spatial coordinate(s), simply because the Higgs configurations vary in this(these) direction(s). In models with both continuous gauge symmetries and discrete symmetries, rich patterns of kink-induced SSB are possible, offering model-building opportunities unavailable in the standard spatially homogeneous case. If spontaneously broken discrete symmetries do not feature in a model of interest, then sufficiently long-lived metastable non-topological kinks are a viable alternative.

One motivation for this paper is to see if this mechanism can help in the construction of GUTs of trinification type. These models have gauge group $SU(3)^3$ and a discrete symmetry permuting the isomorphic $SU(3)$ factors. Another motivation is simply the study of kink solutions in such theories for their own sake, a topic of general interest in field theory, cosmology and condensed matter physics [12].

The present work complements existing clash of symmetries studies in a natural way. The mechanism itself was independently discovered by Davidson, Toner, Volkas and Wali [6] with the brane-world application as motivation, and by Pogosian and Vachaspati [13] in the context of $3 + 1$-dimensional GUT theories. Davidson et al constructed a toy model that essentially consisted of three $SU(3)$ Higgs triplets with a discrete symmetry under permutations, while Pogosian and Vachaspati focused on adjoint kinks in $SU(N)$ models, especially $SU(5)$. Here we will study Higgs fields in $(\mathbf{3}, \overline{\mathbf{3}})$ representations, thus naturally extending the Davidson et al scenario. It is also related in an interesting way to adjoint kinks, because, under the diagonal subgroup, $(\mathbf{3}, \overline{\mathbf{3}})$ decomposes into a (complex) adjoint and singlet. Furthermore, the Davidson et al model was a distillation of ideas fermenting around the GUT group E_6, which has the trinification GUT group $SU(3)^3$ as a subgroup [14]. The other GUT subgroup of E_6, namely $SO(10)$, has already been partially analysed from the clash of symmetries perspective [7], and some quite encouraging SSB outcomes were obtained. One reason for embarking on the present study was to see if the alternative trinification route from an underlying E_6 might also lead to promising model building opportunities.

In the next section, we review trinification and the clash of symmetries idea. Following that, we show that the one-Higgs-multiplet model cannot be made to work even with kink-induced symmetry breaking. In Sec. IV we outline a novel way to avoid the intra-multiplet VEV hierarchies required in standard trinification models. We conclude in Sec. V.

II.   TRINIFICATION, SYMMETRY BREAKING AND THE CLASH OF SYMMETRIES

A.   Trinification and standard symmetry breaking

The trinification gauge group is

$$G_3 = SU(3)_c \otimes SU(3)_L \otimes SU(3)_R, \tag{2.1}$$

where c stands for colour, and $L(R)$ for left(right). One family of quarks, leptons and exotic partner fermions Ψ_L is placed in the anomaly-free, reducible 27-dimensional representation

$$\Psi_L \sim \underline{\mathbf{27}} = \left(\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}\right) + \left(\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}\right) + \left(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}\right). \tag{2.2}$$

Gauge coupling constant unification is achieved by imposing a discrete Z_3 symmetry that permutes the $SU(3)$ factors.

The physical identification of the components in the multiplets depends on the identification of the electric charge, equivalently hypercharge, generator. It is interesting to note that there are three possible choices, that is, there are three embeddings of electric charge and hypercharge in the trinification gauge group. The same is also true in E_6, as has been commented on several times in the literature (note that E_6 and G_3 have the same rank) [14].

An interesting feature of trinification (and E_6) models is that the Higgs multiplets Φ coupling to the fermion bilinears transform the same way as the fermions,

$$\Phi \sim \left(\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}\right) + \left(\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}\right) + \left(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}\right), \tag{2.3}$$

making two independent invariant Yukawa terms of the form $\overline{(\Psi_L)^c}\Psi_L\Phi$. Previous work [2, 3] has shown that at least two copies of such a Higgs multiplet are needed to achieve realistic spontaneous symmetry breakdown and fermion mass generation in the context of ordinary $3 + 1$ dimensional gauge theory. More recently, Willenbrock [4] has observed that gauge coupling constant unification will happen in non-supersymmetric trinification models if a sufficient Φ multiplicity is introduced. This is an interesting observation, even though the resulting model apparently must suffer from the gauge hierarchy problem. We shall return to the trinification gauge hierarchy issue in a later section. For the moment, let us focus on the two-Φ scenario, studied in some depth in Ref.[3].

Let the two Higgs multiplets be called Φ and χ, and denote the irreducible components as per

$$\phi_c \sim \left(\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}\right), \quad \phi_L \sim \left(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}\right), \quad \text{and} \quad \phi_R \sim \left(\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}\right), \tag{2.4}$$

and similarly for χ. Each irreducible piece is best represented by a 3×3 matrix. The transformation law for the colourless field ϕ_c is then $\phi_c \to U_L \phi_c U_R^\dagger$ where $U_{L,R}$ are $SU(3)_{L,R}$ matrices, and so on. We shall denote the generators of $SU(3)_L$ by $L_{1,\dots,8}$, similarly for the right-handed group.

A VEV of the form

$$\langle\phi_c\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v \end{pmatrix} \tag{2.5}$$

spontaneously breaks $SU(3)_L \otimes SU(3)_R$ to a standard left-right symmetric subgroup $SU(2)_L^{(1,2)} \otimes SU(2)_R^{(1,2)} \otimes U(1)_{B-L}$, where

$$B - L = L_8 + R_8. \tag{2.6}$$

The $SU(2)$ factors act on the top-left 2×2 block: the superscript $(1,2)$ means 'acting on the first and second rows and columns' for the left- and right-handed groups respectively.

Suppose the second multiplet develops a VEV of the form

$$\langle\chi_c\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & v' & 0 \end{pmatrix}. \tag{2.7}$$

The breakdown induced by this VEV on its own is also to a left-right symmetric subgroup, namely $SU(2)_L^{(1,2)} \otimes SU(2)_R^{(1,3)} \otimes U(1)_{Y''}$. The right-handed $SU(2)$ is now differently embedded within the parent $SU(3)_R$, and the Y'' generator is

$$Y'' = L_8 - 2R_8, \tag{2.8}$$

the third embedding of hypercharge. The effect of the two VEVs acting together is to break $SU(3)_L \otimes SU(3)_R$ to the *intersection* of the two, differently-embedded left-right symmetric subgroups defined above. The intersection group is precisely the electroweak group $SU(2)_L^{(1,2)} \otimes U(1)_Y$, where

$$Y = R_3 + L_8 + R_8. \tag{2.9}$$

It is interesting to note that intersections of differently embedded but isomorphic subgroups is also central to the clash of symmetries mechanism.

The other two hypercharge embeddings are obtained by permuting the nonzero entries along the bottom row of the VEV matrices: $Y' = L_8 - R_3 + R_8$ requires them to be in the first and third slots, while Y'' needs the first and second slots.

To complete our discussion, we address electroweak symmetry breaking. For definiteness, we focus on the hypercharge embedding of Eq. (2.9). Using the definition of electric charge

$$Q = \frac{1}{2} \left(L_3 + Y \right), \tag{2.10}$$

we see that, up to gauge transformations, the most general VEV patterns inducing complete breakdown to $U(1)_Q$ are

$$\langle \phi_c \rangle = \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & v_1 \end{pmatrix}, \qquad \langle \chi_c \rangle = \begin{pmatrix} u_3 & 0 & 0 \\ 0 & u_4 & u_5 \\ 0 & v_2 & v_3 \end{pmatrix}, \tag{2.11}$$

where the u's are of order the electroweak scale ($\ll v_{1-3}$). Reference [3] explains why having just one multiplet, whose VEV we can take to be of the form $\langle \phi_c \rangle$ above without loss of generality, leads to unrealistic fermion masses and mixings. Unfortunately, the most general quartic potential with both Φ and χ has very many independent terms and parameters. This goes against the fundamental reason for introducing larger gauge symmetries: the hope of greater predictivity. Furthermore, one requires a delicate VEV hierarchy *within each multiplet*, $u_{1-5} \ll v_{1-3}$. Needless to say, this necessitates unnatural fine-tuning.

B.   Kink-induced symmetry breaking

Topologically stable or metastable kinks serve as dynamically-induced branes, and they in general feature symmetry breaking patterns or hierarchies that depend on the extra dimension coordinate w.

When a symmetry group G spontaneously breaks to a subgroup H, the vacuum manifold is usually given by the coset space G/H. All elements of G/H yield isomorphic stability groups, but different elements imply different embeddings of the subgroup H within G. When the parent group has a discrete group factor that is spontaneously broken, the vacuum manifold is disconnected. It contains a discrete number of copies of G/H (where G is the continuous part of the parent group).

By definition, kinks are static one-dimensional solutions of the classical Euler-Lagrange equations that have as boundary conditions that the configurations asymptote to elements of the vacuum manifold as $w \to +\infty$ and $w \to -\infty$. If these two elements live on disconnected pieces, then the kink is topologically stable. If not, it is non-topological but possibly metastable or perturbatively stable. This depends on the Higgs potential topography: for perturbative stability, there must be an initial energy cost when perturbations to the kink are added.

The simple Z_2 kink used in pedagogical discussions arises in a model where the vacuum manifold is simply two (disconnected) points. The features we wish to explore occur when G/H is a non-trivial manifold, because now there are many choices for the kink boundary conditions.

The pure clash of symmetries mechanism arises when the kink begins and ends on disconnected pieces of the vacuum manifold such that the stability groups at $w = +\infty$ and $w = -\infty$ are differently embedded (though isomorphic) H subgroups of G. We shall call these groups $H(+\infty)$ and $H(-\infty)$. If, by contrast, they are identically embedded $[H(+\infty) = H(-\infty) \equiv H]$, then the symmetry group at most points w is just the very same H. With different embeddings asymptotically, the kink has to perform a non-trivial interpolation. This means that, in general, the unbroken symmetry at points $|w| < \infty$ is the intersection $H(+\infty) \cap H(-\infty)$, so it is smaller than H. The possible advantage here is greater symmetry breaking power from a given Higgs model compared to the standard case where a spatially homogeneous VEV configuration is used. Note that there may be special points, most commonly $w = 0$, where the configuration instantaneously passes through a special breaking pattern. This arises if some of the components of

a multiplet are odd functions of w (such as the hyperbolic tangent), because at $w = 0$ those components vanish and symmetry breaking is reduced.

Higgs multiplets in models with non-trivial G/H manifolds are multi-component. When we say "kink configuration" in this context, we do not mean that all nonzero components have to be odd functions like the standard Z_2 kink. The components may be odd functions, or even functions, or neither, and these may coexist within the one multiplet. It depends on the specific VEVs to which the configuration asymptotes. Consider a given component ϕ_i of a multiplet Φ. An odd function for that component arises if $\phi_i(+\infty) = -\phi_i(-\infty)$, an even function if $\phi_i(+\infty) = +\phi_i(-\infty)$, and neither if $\phi_i(-\infty) \neq 0$ but $\phi_i(+\infty) = 0$ (or vice-versa). This has an interesting consequence. Even if $H(+\infty) = H(-\infty) = H$ so that the symmetry breaking is almost always to the same H, relative sign changes in the asymptotic components of Φ can mean that intra-multiplet hierarchies change as a function of w. In particular, we can hope to exploit the differences between odd- and even-function components to generate a large hierarchy in effective VEVs near the brane at $w = 0$. We will use this possibility in Sec. IV.

III.   SYMMETRY BREAKING AND KINKS IN $SU(3) \otimes SU(3)$ WITH ONE $(\mathbf{3}, \overline{\mathbf{3}})$ HIGGS MULTIPLET

Symmetry breaking in trinification models is greatly concerned with VEV patterns for Higgs fields in $(\mathbf{3}, \overline{\mathbf{3}})$ representations of an $SU(3)_1 \otimes SU(3)_2$, identified with the extended left-right group in those models. This provides good motivation for studying Higgs potentials, symmetry breaking patterns and kink solutions for a model with a single Higgs field $\Phi \sim (\mathbf{3}, \overline{\mathbf{3}})$. This is also of general interest because it is a rich theoretical laboratory despite its apparent simplicity. It ties in well with studies of $SU(N)$ adjoint Higgs models since, as already mentioned, Φ branches into a complex octet and singlet under the diagonal $SU(3)$ subgroup.

The field Φ is represented as a general 3×3 matrix of complex field components,

$$\Phi = \left(\phi_i^j \right), \qquad \left(\phi_i^j \right) \in \mathfrak{C}, \qquad i, j = 1, 2, 3, \tag{3.1}$$

undergoing the transformation

$$\Phi \rightarrow U_1 \Phi U_2^\dagger, \tag{3.2}$$

where $U_{1,2}$ are elements of the fundamental representations of $SU(3)_{1,2}$, respectively.

A.   The quartic potential model

We studied a model described by a quartic Φ potential in the most detail as quartics are the simplest non-trivial potentials which can be bounded from below, and they are renormalisable in $3 + 1$ dimensions. (For now we ignore the question of ultraviolet completion in higher dimensional models, leaving it as a problem for the future).

The most general quartic $SU(3)_1 \otimes SU(3)_2$ invariant potential is

$$V = -m^2 Tr[\Phi^\dagger \Phi] + \lambda_1 \left(Tr[\Phi^\dagger \Phi] \right)^2 + \lambda_2 Tr[\Phi^\dagger \Phi \Phi^\dagger \Phi] + (a \, \Phi_i^l \Phi_j^m \Phi_k^n \epsilon^{ijk} \epsilon_{lmn} + H.c), \tag{3.3}$$

which is bounded from below in the parameter region

$$\lambda_1 > 0, \qquad \text{and} \qquad 3\lambda_1 + \lambda_2 > 0. \tag{3.4}$$

If $a = 0$, then the potential has an extra $U(1)$ symmetry $\Phi \rightarrow e^{i\theta} \Phi$. With $a \neq 0$, this $U(1)$ is explicitly broken to the discrete Z_3 subgroup.

For homogeneous field configurations, an $SU(3)_1 \otimes SU(3)_2$ transformation can always be used to bring Φ into diagonal form,

$$\Phi = e^{i\alpha} \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{pmatrix}, \tag{3.5}$$

where $\alpha, \phi_i \in R$. The potential now possesses invariance under a permutation symmetry S_3 also.

Global minimisation revealed four non-trivial extrema that are global minima for certain ranges of the Higgs parameters, where these regions of parameter space were studied extensively. Although this analysis is important in the study of kinks, we do not show it here as it was found that the one-Higgs-field model showed no promise in model-building outside the context of the pure study of kinks. Rather, we indulge in a brief summary of our results.

The symmetry breaking patterns instigated by the four global minima leave the unbroken subgroups $SU(2)_1 \otimes SU(2)_2 \otimes U(1)$, $SU(3)_{1+2}$, and $SU(2)_{1+2} \otimes U(1)$. Clearly, a single Higgs field in conventional trinification models can not provide sufficient symmetry breaking capabilities unless we invoke kink-induced symmetry breaking.

Invariance of our potential under the discrete symmetries S_3, Z_3 means that each VEV is actually one choice out of a set of vacua which yield degenerate global minima. These represent our coset spaces G/H which form our vacuum manifold. Thus, we can obtain VEV configurations which leave differently embedded isomorphic subgroups unbroken, and choose these to be the boundary conditions of our kinks. At non-asymptotic points, the kink will then respect a reduced symmetry given by the intersection of the two groups.

Given the degeneracy of our vacua and the wealth of their parameter space dependence, there are a large number of distinct kinks which can be constructed. In section II B it was outlined how the stability of these domain wall solutions depends on the discrete symmetry Z of the theory. Topologically non-trivial kinks asymptote to vacua which are elements of different connected sectors, each being G/H, while the spontaneously broken group elements of Z map each sector into one another. As a result of this broken discrete symmetry, there is an energy barrier between the two vacua, ensuring the stability of the kink. However, in this model, the question of stability is not so explicitly defined. The discrete transformations of our quartic potential, Z_3, S_3, are contained in $SU(3)_1 \otimes SU(3)_2$ and so cannot assume the privileges of Z. Instead, the kinks will only lie in disconnected coset spaces if they cannot be connected by any continuous symmetry of our Higgs potential. That is, for any elements $U_1 \in SU(3)_1$ and $U_2 \in SU(3)_2$,

$$\langle\Phi\rangle_1 \not\rightarrow U_1\langle\Phi\rangle_2 U_2^\dagger. \tag{3.6}$$

For the special case of $a = 0$, the Higgs potential has an additional $U(1)$ symmetry also via which the vacua should not be related. If any two VEV configurations can be connected by any one of these continuous transformations then they lie in the same sector G/H. Kinks in this instance are non-topological, though they might be metastable and sufficiently long-lived to be useful.

Unfortunately, for all possible cases, it is trivial to find either a $U_{1,2}$ such that the equality of Eq. (3.6) holds, or a $U(1)$ connecting the VEVs. So, although it has a rich family of non-topological configurations, the quartic potential one-Higgs-field model does not admit any topological kinks.

We conclude this section by noting that these configurations are interesting in their own right, but they are not of direct applicability for the trinification symmetry breaking problem: almost all of the unbroken $SU(2)$ factors are diagonal subgroups, leaving only vector-like combinations unbroken with the identification of $(1, 2)$ with (L, R).

B. Beyond the quartic potential

We now study what additional possibilities open up if we consider non-quartic potentials. In order for a spatially varying Higgs field to be viable, we must associate ourselves with a brane-world hypothesis, where the Higgs field propagates in the bulk. This complicates the question of ultraviolet completeness. Lacking much phenomenological guidance on its structure, we are a priori free to choose any bounded Higgs potential. A sextic potential for example, would contain terms proportional to $(\det\Phi)^2$ which break the $U(1)$ to a Z_6 symmetry which is not a subgroup of $SU(3) \otimes SU(3)$. Thus, if this Z_6 symmetry was spontaneously broken then the stability of our kinks would be guaranteed. So it is possible to find topological kinks for a one field model for non-quartic Higgs potentials.

It may also be fairly straightforward to construct non-quartic Higgs potentials such that non-topological kinks become at least perturbatively stable. As part of work in progress, D.P. George and one us (RRV) [15] has demonstrated that sextic potentials with a sufficiently deep local (not global) minimum at $\phi = 0$ will produce perturbatively stable kinks interpolating between $\phi = v$ and $\phi = -v$, where v is the value of ϕ at the global minimum. The toy model

studied here was of a single complex scalar field ϕ with a $U(1)$ invariant sextic potential. We expect that this result can be extended to non-Abelian multi-component Higgs models, though this has not as yet been rigorously checked.

Now, it is not necessary to write down a specific potential in order to determine the qualitative pattern of symmetry breaking as a function of the bulk coordinate. To determine the VEVs that can serve as boundary conditions, independent of the potential, we need only to write down all possible diagonal configurations for our single Higgs multiplet, as per Eq. (3.5). This yields the unbroken subgroup $U(1) \otimes U(1)'$ as a possible symmetry breaking avenue in addition to those of the quartic potential case.

Consider now any general Higgs potential which is $SU(3)^2$ invariant and assume that it also has a discrete symmetry which, firstly, is not contained in $SU(3)^2$, and secondly, is not a subgroup of any additional accidental continuous symmetries. We wish to find all possible pairs of VEVs which are transformed into each other by discrete transformations outside $SU(3)^2$. They are

$$\Phi(-\infty) = \langle\Phi\rangle_7 = diag\,(0, a, b)\,, \qquad \Phi(+\infty) = \langle\Phi\rangle_8 = diag\,(0, -a, b)\,, \quad + \text{ cyc. perms,} \tag{3.7}$$

$$\Phi(-\infty) = \langle\Phi\rangle_9 = diag\,(a, b, c)\,, \qquad \Phi(+\infty) = \begin{cases} \langle\Phi\rangle_{10} = diag\,(-a, b, c)\,, \quad + \text{ cyc. perms} \\[2mm] \langle\Phi\rangle_{11} = diag\,(-a, -b, -c)\,, \end{cases} \tag{3.8}$$

where a, b and c are not necessarily unequal.

We now give an example of the kind of topological kink that can arise in a non-quartic model. The most interesting case has kinks interpolating between the VEVs of Eq. (3.7). Let $\Phi(-\infty) = \langle\Phi\rangle_7$, and $\Phi(+\infty) = \langle\Phi\rangle_8$, then $\Phi(w) = diag\,(0, -a\,f(w), b)$, where $f(w)$ is some kink-like odd function (perhaps a hyperbolic tangent) whose specific form of course depends on the Higgs potential chosen. Independent of this choice, we see that the w-dependent symmetry breaking pattern has $H(-\infty) = H(+\infty) = H_{clash} = U(1) \otimes U(1)'$, but on the brane, at $w = 0$, $\Phi(0) = diag\,(0, 0, b)$ and the symmetry is enhanced to $H(0) = SU(2)_1 \otimes SU(2)_2 \otimes U(1)$. We see that while the left-right symmetry group is exact at $w = 0$, the additional breakdown off the brane is quite strong, to $U(1) \otimes U(1)'$.

We conclude that a rich family of topologically stable kinks may be produced by models with non-quartic Higgs potentials. This is interesting, though from the point of view of symmetry breaking in trinification models none of the additional configurations possible for non-quartic potentials seem to be of direct use, as they generate the wrong breaking patterns.

IV. TRINIFICATION WITH TWO $(\mathbf{3}, \overline{\mathbf{3}})$ HIGGS MULTIPLETS

A. Application to the trinification hierarchy problem

We have seen that the one-field model is an interesting theoretical laboratory in terms of VEV patterns, associated kinks (either topological or not depending on the case) and symmetry breaking patterns. However we did not find any case that was obviously promising for the trinification application, even when kink-induced symmetry breaking was used. We thus now turn to the two field case.

Reiterating the introductory discussion, the trinification gauge group is given by

$$G_3 = SU(3)_c \otimes SU(3)_L \otimes SU(3)_R, \tag{4.1}$$

which is augmented by the cyclic symmetry Z_3 to ensure a single gauge coupling constant. The symmetry breaking to the SM gauge group is compactly accomplished with the VEV configuration of Eq. (2.11) where $u \sim 10^2 GeV$ and $v \sim 10^{15} GeV$. The gauge hierarchy and fine-tuning problems in this model are quite delicate as there are hierarchies in the VEVs within a single multiplet, there being entries of both the unification and electroweak breaking scales. We will see that the introduction of the spatial variation in the Higgs field may be able to alleviate the hierarchy problem in a novel way. We restrict ourselves to considering only two colour-singlet Higgs fields ϕ_c, χ_c which transform under Eq. (2.1) as ϕ_c, $\chi_c \sim (1, \mathbf{3}, \overline{\mathbf{3}})$.

The two colour-singlet Higgs fields are allowed to have the most general spatial variation in the extra dimensional coordinate,

$$\phi_c(w) = \begin{pmatrix} \varphi_1(w) & 0 & 0 \\ 0 & \varphi_2(w) & 0 \\ 0 & 0 & \varphi_3(w) \end{pmatrix}, \qquad \chi_c(w) = \begin{pmatrix} \chi_1(w) & 0 & 0 \\ 0 & \chi_2(w) & \chi_3(w) \\ 0 & \chi_4(w) & \chi_5(w) \end{pmatrix}, \qquad (4.2)$$

but we relax the constraint that they leave differently embedded, isomorphic subgroups unbroken at $|w| = \infty$ as would be demanded if we were considering the "pure" clash idea. The VEVs retain the structural form of Eq. (2.11), however all entries are elevated to be of the unification order. With this modification, the model now admits only one SSB scale, eliminating the hierarchy. We will see that a hierarchy in observed symmetry breaking scales may nevertheless be effectively generated through the details of the spatial variation of the various components within ϕ_c and χ_c.

At the asymptotic points $|w| = \infty$, the Higgs fields must approach VEV profiles which are related to each other via a discrete symmetry of the potential. We do not wish to specify a Higgs potential, but rather complete the most generalised analysis possible. So we consider all the VEV configurations which may be related by a discrete symmetry of some potential. The most general set of discrete transformations are independent reflection symmetries in each individual field component. Requiring these VEVs to be disconnected from each other greatly restricts ϕ_c, giving only four possible choices. If $\phi(-\infty) \equiv \phi_v^1 = diag\,(u_1, u_2, v_1)$, then

$$\phi_c(+\infty) \equiv \phi_v^2 = diag\,(-u_1, u_2, v_1), \qquad \phi_c(+\infty) \equiv \phi_v^3 = diag\,(u_1, -u_2, v_1),$$
$$\phi_c(+\infty) \equiv \phi_v^4 = diag\,(u_1, u_2, -v_1) \quad \text{and} \quad \phi_c(+\infty) \equiv \phi_v^5 = diag\,(-u_1, -u_2, -v_1).$$

The field χ_c is much less restricted. There are a large number of distinct combinations of VEVs which are not connected by an $SU(3)^3$ transformation. We can narrow this set by observing the behaviour of the field on the brane. The profile of the field on the brane is determined by considering the parity of the configurations which interpolate between these VEVs. That is, we can ascertain which components pass through zero on the brane without the explicit derivation of the form of the kinks. If the χ_1^1 component is non-zero on the brane, then the $SU(2)_L^{(1,2)}$ subgroup will be broken. However, this is exactly the weak isospin gauge group of the Weinberg-Salam model which must remain unbroken for realistic model-building. So we need only consider the VEVs which respect $\chi_1(-\infty) = -\chi_1(+\infty)$, reducing the number of choices. Of these, there are only 20 distinct configurations on the brane, and given that it is the symmetry breaking on the brane with which we are concerned, we can focus our analysis on these without loss of generality.

Together, with the possibilities for ϕ_c, there are 80 unique possible kink configurations for a model described by the trinification gauge group. Consider first the symmetry breaking initiated by ϕ_c. If $\phi_c(-\infty) = \phi_v^1$ and $\phi_c(+\infty) = \phi_v^2$, ϕ_v^3 or ϕ_v^4, then $\phi_c(0)$ has two non-zero entries on the brane. The best case scenario for these choices will give a maximum unbroken subgroup of $SU(2)_L \otimes SU(2)_R \overset{\phi_c(0)}{\to} SU(2)_{L+R} \otimes U(1)$. Hence, when the intersection is taken with the appropriate χ profile, the largest unbroken symmetry possible on the brane is $SU(3)_c \otimes SU(2)_{L+R} \otimes U(1)$, subject to specific parameter constraints. All other instances have the chiral $SU(3)$'s break to $U(1)$'s. This is clearly insufficient in the model-building perspective because weak isospin is vector-like.

As a result, there is a unique choice for $\phi_c(w)$, given by $\phi(-\infty) = \phi_v^1$ and $\phi(+\infty) = \phi_v^5$, leaving only 20 distinct configurations on the brane to consider. As $\phi_v^5 = -\phi_v^1$, all of the components of $\phi_c(w)$ pass through zero at $w = 0$, leaving $\chi(0)$ solely responsible for all the breaking on the brane. Analysing the $\chi_c(w)$ configurations, there is a unique pattern of symmetry breaking on the brane that offers merit. This is given by the boundary conditions

$$\chi_v^1 \equiv \chi_c(-\infty) = \begin{pmatrix} u_3 & 0 & 0 \\ 0 & u_4 & 0 \\ 0 & v_2 & v_3 \end{pmatrix}, \qquad \chi_v^2 \equiv \chi_c(+\infty) = \begin{pmatrix} -u_3 & 0 & 0 \\ 0 & -u_4 & 0 \\ 0 & v_2 & -v_3 \end{pmatrix}, \qquad (4.3)$$

where we have performed an $SU(2)_R$ transformation on χ_c to rotate u_5 to zero. Despite the rotation, these boundary conditions are still disconnected via an $SU(3)^3$ transformation, with the rotation chosen in order to obtain the correctly embedded $SU(2)_L$ subgroup.

On the brane,

$$\chi(0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & v_2 & 0 \end{pmatrix} \tag{4.4}$$

which instigates the breaking

$$SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \to SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y''}. \tag{4.5}$$

Enforcing the brane world hypothesis realistically to this model may provide a novel way to understand the gauge hierarchy as first explored in [7]. The assignment of w as the extra dimensional coordinate, enables the Higgs configuration to propagate through the extra dimensions. The brane is located at $w = 0$, on which the SM degrees of freedom are confined. Actually the degrees of freedom for the brane world theory are larger than the SM. The theory localised to the brane is the full modified trinification theory described above, its Lagrangian described by the symmetry of G_3. The brane world fields then interact with the Higgs fields ϕ_c, χ_c, that instigate symmetry breaking.

Recall that our VEV entries are all of the unification scale, so as the fields vary spatially in w, their components range from $0 - 10^{15}GeV$ in a natural way. On the brane, $\phi_c(0) = 0$ and $\chi_c(0)$ has one non-zero component $v_2 \sim 10^{15}GeV$, and this Higgs field *strongly* induces the breakdown of Eq. (4.5) to a left-right symmetric group. It is difficult to believe that our trinification theory has a definite, sharp localisation at $w = 0$. If the brane world degrees of freedom are permitted a slight leakage off the wall, then the effective brane world theory will also be affected by the Higgs field for small, finite values of w centred about the brane. This instigates an additional symmetry breakdown, but at a much *weaker* scale, the strength being proportional to the amount of leakage, and equivalently, the distance scale over which the Higgs field components substantially vary. At these points, the Higgs configurations are of the form of Eq. (4.2). Individually, each field breaks the left-right symmetry, the intersection of their unbroken subgroups determining the symmetry in the bulk. ϕ_v^5 generates the breaking

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y''} \to SU(3)_c \otimes U(1)_{L_3+R_3} \otimes U(1)_{L_8+R_8}, \tag{4.6}$$

while χ_v^1 is responsible for the breaking

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y''} \to SU(3)_c \otimes U(1) \otimes U(1)'. \tag{4.7}$$

Together, the two multiplets induce the breaking down to the colour and QED gauge groups

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y''} \to SU(3)_c \otimes U(1)_Q. \tag{4.8}$$

So we have 'naturally' achieved electroweak breaking without having to introduce additional, unnatural VEV components in the Higgs fields, and/or additional Higgs fields. In this context, the gauge hierarchy issue which plagues SSB can be attributed to the structural form of the kink solutions, with emphasis on their spatial variation, and the characteristics of the leakage off the brane. It is the amount of leakage and the value of the Higgs field at this finite w which determines the scale of the electroweak breaking. If a well-motivated brane localisation mechanism can be found that allows exponentially small leakage, then the gauge hierarchy would be explained.

With the exception of the degrees of freedom which enjoy the leakage, if the observable 3+1 dimensional universe is confined to this brane, then the symmetry group is not the SM, but a left-right symmetric theory. Although Eq. (4.8) represents a necessary relationship for the model to be compatible with observed electroweak breaking, there is the need for the pattern

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y''} \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \tag{4.9}$$

to be an intermediary. (The hypercharge generator of the SM group here has to be the conventional embedding so as to be consistent with the breaking of Eq. (4.8)). The SM is an established effective field theory at low energies and we must be able to generate its gauge group along this symmetry breaking avenue. It is not sufficient to have only

the left-right symmetric theory of Eq. (4.5) on the brane, with the breaking of the chiral $SU(2)_R$ and electroweak symmetry occurring at the same stage, and hence same energy, in the bulk. Left-right symmetric theories, realised with a $U(1)_{B-L}$ factor, have been extensively studied, with the breaking $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ currently having a phenomenological lower bound of $\sim 10^3$ GeV, whereas the electroweak breaking occurs at $M_{EW} \sim 10^2 GeV$. Therefore the two symmetry breaking stages

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y''} \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \tag{4.10}$$

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q \tag{4.11}$$

may represent a difference in scale by a factor of no more than ten. The form of the kink configurations provides a possible solution. If $\varphi_3(w)$ has a slightly steeper gradient as the ϕ_c Higgs profile approaches the brane, then, to leading order, Eq. (4.9) would occur. The factor of ten may have its origins in a small spread in the Higgs parameters. Subsequently, the total symmetry breaking pattern produced may be

$$SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y''} \rightarrow G_{SM} \rightarrow SU(3)_c \otimes U(1)_Q. \tag{4.12}$$

This two Higgs model based upon trinification theory, therefore, has shown the potential for kink profiles that generate a phenomenologically acceptable symmetry breaking pattern. Furthermore, these kinks and the breaking for which they are responsible, have been constructed without reference to any specific Higgs potential. This means that the question of the existence and stability of our solution reduces to finding an appropriate potential.

As the above shows, the brane-world kink method holds promise for inducing the correct symmetry breaking patterns necessary for trinification models. However, our analysis is obviously incomplete. The main unresolved issues are the incorporation of gravity and the localisation of fields to the brane. The former is likely to be relatively straightforward, but the latter presents some important challenges. The leakage scenario itself is plausible, and inevitable in the case of dynamical localisation where wave-functions are sharply peaked around the brane but not delta-function-like. For the case of one extra dimension, it is known that several ingredients are needed to achieve dynamical localisation of fermions, Higgs fields (if that is necessary) and gauge fields [16]. To conclusively demonstrate that the ideas presented above, which we think are interesting and novel, can be the basis for a realistic brane-world model requires considerably more work and is beyond the scope of this paper.

V. CONCLUSION

Motivated by the attractive trinification route to grand unification, we have examined symmetry breaking patterns and kink solutions in models with one or two Higgs multiplets assigned to the $(\mathbf{3}, \overline{\mathbf{3}})$ representation of an $SU(3)_1 \otimes SU(3)_2$ gauge symmetry.

For the one field case, a rich pattern of symmetry breaking outcomes was rigorously deduced for the case of a quartic Higgs potential. Employing the resulting global minima as boundary conditions, we studied the possible kink configurations of the model. We found that there are many possible non-topological kink configurations, but none that are topologically stable. One motivation for this analysis was to see if the clash of symmetries mechanism of Refs. [6] and [7] and generalisations thereof might be of use in a brane-world realisation of trinification, with the Higgs field propagating in the bulk. Unfortunately, none of the bulk-coordinate-dependent symmetry breaking patterns identified appear to be useful in this regard. We discussed how topologically stable kinks can arise when non-quartic potentials are used, recognising that the restriction to quartic form is not mandatory for brane-world models with their unknown ultraviolet completion. The strategy here was to identify those discrete transforms of Higgs global minima configurations that are outside the continuous $SU(3)_1 \otimes SU(3)_2$ gauge group, and to use the spontaneous breaking of those discrete invariances to ensure topological stability. The assumption here was that one could construct a non-quartic Higgs potential so as to realise the discrete symmetries required.

We then turned to the two-field case in the context of the brane-world trinification application. In the usual $3 + 1$-dimension incarnation of the model, two $(\mathbf{3}, \overline{\mathbf{3}})$ Higgs multiplets are required to ensure phenomenologically acceptable

symmetry breaking and fermion mass generation. However there is a severe fine-tuning problem: some components of each multiplet must be given unification scale values, while others must be of electroweak scale. We put forward what we think is an interesting, albeit speculative, alternative: Go to a brane-world realisation with one extra dimension, and have *all* nonzero components of the Higgs fields acquire unification-scale values only. Then effectively induce the electroweak to unification scale hierarchy by exploiting the spatial dependence of the Higgs components with respect to the bulk coordinate. Those components that are odd functions (say of hyperbolic tangent form) go to zero exactly on the brane, despite the fact that they asymptote to unification scale values infinitely far from the brane. By exploiting these zeros and postulating mild differences in the slopes of the kinked components near the brane, we found a situation where a hierarchical cascade, trinification to left-right symmetry to the standard model to just colour cross electromagnetism, might ensue.

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